IMPINGEMENT OF A SHOCK WAVE ON A WEDGE MOVING AT SUPERSONIC SPEED

(NABEGANIE UDARNOI VOLNY NA KLIN, DVIZHUSHCHIISIA SO SVERKHZVUKOVOI SKOROST'IU)

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If a plane shock wave impinges at a certain angle on the front of a body moving through a gas at supersonic speed, upon approaching the body it first interacts with the bow shock wave ahead of the body, and there develops the whole complicated pattern of diffracted shock waves around the moving body, accompanied in the general case by a series of new reflected shock waves and contact discontinuities.

However it is found that there exist particular cases in which this pattern is very simple and easily calculated.

We consider the upper surface of a wedge of half angle δ moving at supersonic speed V with an attached bow shock wave. Let a plane shock wave inclined at angle β to the vertical propagate with speed c toward the front of the wedge.

We show that if the parameters V, δ, c and β are subject to certain conditions, then on the upper surface of a wedge with an impinging shock wave the flow picture will correspond at a certain instant to that depicted in Fig.1, where BA and BD are the parts of the attached bow wave and the impinging shock wave that are still not undergoing interaction, BO is the new attached bow wave arising after interaction, and BC is the continuation of the impinging shock wave. We remark that in a system of coordinates with origin at the vertex of the wedge the picture is self-similar in time (that is, depends on the ratios of the coordinates to the time). In region OBC the flow has constant parameters. We consider the conditions to be imposed on V, δ, c and β in order that this flow be realized.

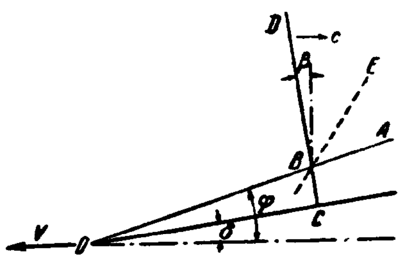

Fig. 1

First, the intensities (that is, differential pressures) of the impinging and attached shock waves must be equal in order to avoid a contact discontinuity from point B in region BOC. Then the intensities of shock waves OB and BC are also equal. Second, the line BO must be the continuation of AB, which is evident from geometrical considerations. Now introducing the bisector BE and connecting the origin of coordinates with point B, it is easy to see that such a configuration of shock waves arises through reflection of a shock from the wall

BE under the condition that the angle of incidence ω is equal to the angle of reflection ω_1. It is known (for example, from [1]), that reflection of a shock wave of finite intensity in this way is possible only for a definite angle of incidence ω^*, equal to

$$\omega^* = \cot^{-1}[(\gamma+1)/(3-\gamma)]^{1/2} \quad (\gamma \text{ is the ratio of specific heats}) \tag{1}$$

for arbitrary intensity of the incident shock wave. The relative differential pressure p° in the reflected shock wave is found in this case just as for a frontal encounter with a wall, and is related to the relative differential pressure in the incident wave by Formula

$$p^\circ = \frac{(3\gamma-1)\,p-(\gamma-1)}{(\gamma-1)\,p+(\gamma+1)} \tag{2}$$

Thus in order to realize the flow depicted in Fig.1 it is necessary that the impinging and bow shock waves have equal intensity and intersect at an angle equal to $2\omega^*$.

Furthermore, the shock wave BC, being the extension of the shock wave DB in such a shock intersection, must be normal to the surface of the wedge. (This is possible since $2\omega^* < {}^1/_2\pi$ for $1 < \gamma \leqslant 3$; for $\gamma > 3$ such a flow pattern is not possible). From this condition it follows that

$$\beta = \delta \tag{3}$$

Furthermore, it follows from consideration of the triangle OBC that the condition of intersection of the shock waves at angle $2\omega^*$ is equivalent to condition

$$\varphi - \delta = {}^1/_2\pi - 2\omega^* \tag{4}$$

The angle between the attached shock wave and the surface of the wedge must have a fixed value depending only upon γ and equal to 11.54° when $\gamma = 1.4$ (for air).

Thus the four parameters V, δ, φ and β must satisfy three conditions: equality of intensities of the shock waves, and conditions (3) and (4). As a result a family of solutions is obtained depending upon one parameter.

We express condition (4) in the form of a relation between the wedge angle and the Mach number M of the oncoming stream. By virtue of (1) this condition takes the form

$$\tan(\varphi-\delta) = \cot 2\omega^* = \frac{\gamma-1}{\sqrt{(\gamma+1)(3-\gamma)}}, \qquad \text{or} \qquad \tan\delta = \frac{\tan\varphi - \cot 2\omega^*}{1+\tan\varphi\cdot\cot 2\omega^*} \tag{5}$$

On the other hand, from the equations for an oblique shock wave we have

$$\tan\delta = \left[\tan\varphi\left(\frac{\gamma+1}{2}\,\frac{M^2}{M^2\sin^2\varphi-1}-1\right)\right]^{-1} \tag{6}$$

Equating the right-hand sides we obtain, after some manipulation

$$\left[\tan^2\varphi\left(1+M^2\frac{\gamma-1}{2}\right)-\tan\varphi\,\cot 2\omega^* M^2\frac{\gamma+1}{2}+1\right](\tan^2\varphi+1)=0$$

This equation has the following real solutions:

$$\tan\varphi = \left[\cot 2\omega^* \pm \left(\cot^2 2\omega^* - \frac{8(\gamma-1)}{M^2(\gamma+1)^2} - \frac{16}{M^4(\gamma+1)^2}\right)^{1/2}\right] \times$$

$$\times\left[2\left(\frac{\gamma-1}{\gamma+1}+\frac{2}{M^2(\gamma+1)}\right)\right]^{-1} \tag{7}$$

Thus from (7) and (5) it is possible to find for a given Mach number M the shock wave angle and wedge angle for which condition (4) is satisfied. Then for a shock wave impinging on the wedge at angle β equal to the wedge angle and with intensity equal to that of the attached wave, the flow field under consideration is realized on the wedge. It is easy to see from Equation (7) that such a flow exists for

$$M \geqslant 2\left(\frac{3-\gamma+2\sqrt{3-\gamma}}{(\gamma+1)(\gamma-1)}\right)^{1/2} \quad \text{(or for } M \geqslant 4.15 \text{ with } \gamma = 1.4) \qquad (8)$$

Furthermore, it can be shown that for

$$M > \frac{1}{\cot 2\omega^*}\sqrt{\cot^2 2\omega^* + 1} \qquad (9)$$

the second solution (with the minus sign on the radical), has no sgnificance because the shock wave angle is less than the Mach angle.

In the range of Mach number between these limits both solutions (7) have significance.

We observe that as $M \to \infty$ we obtain for the shock wave angle

$$\lim_{M\to\infty} \varphi = \tan^{-1}\left(\frac{\gamma+1}{3-\gamma}\right)^{1/2} = \frac{1}{2}\pi - \omega^*$$

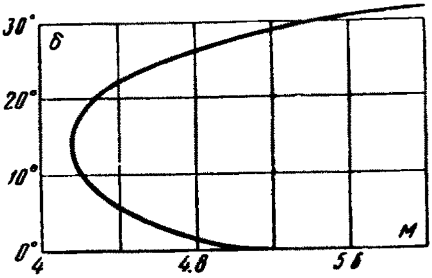

Fig. 2

or, using (4), the limiting value of the wedge angle is found to be equal to the angle of incidence ω^*. We note also that for $\gamma \to 1$ the wedge angle approaches $\frac{1}{4}\pi$, and as γ increases to 3 the wedge angle decreases, vanishing at $\gamma = 3$. As mentioned above, no solution exists for $\gamma > 3$. Fig.2 shows for illustration the values of the wedge angle for various Mach numbers in the case $\gamma = 1.4$.

BIBLIOGRAPHY

1. Mises, R., Matematicheskaia teoriia techenii szhimaemoi zhidkosti (Mathematical Theory of Compressible Fluid Flow). IL, 1961.

Translated by M.D. VanD.